



PANIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

**FACULTY OF HEALTH, APPLIED SCIENCES, AND NATURAL
RESOURCES**

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION : BACHELOR OF SCIENCE APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: 07BAMS	LEVEL: 5
COURSE: PROBABILITY THEORY 1	COURSE CODE: PBT501S
DATE: JUNE 2022	SESSION: JUNE
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Dr. D. Ntirampeba Mr. E. Mwahi
MODERATOR:	Mr. A. Roux

THIS QUESTION PAPER CONSISTS OF 3 PAGES
(Excluding this front page and statistical tables)

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programable calculator

ATTACHMENTS

1. Statistical tables (Z-Tables)

QUESTION 1 [30 Marks]

1.1. Consider the events $A = \{1,3\}$, $B = \{2,5\}$, $C = \{6\}$, $D = \{3,4\}$ over a possibility space $S = \{1,2,3,4,5,6\}$. With reasons, state whether true or false:

1.1.1 Events A and C are mutually. [2]

1.1.2. $\{A, B, C, D\}$ is a partition of the sample space S [3]

1.1.3. $\emptyset^c = \{1,2,3,4,5\}$ [2]

1.1.4. $B \Delta C = \{1, 3\}$ [3]

1.2. Let A be an event in a sample S. Show that $P(A^c) = 1 - P(A)$ [4]

1.3. Let A and B be events in a sample S . Show that $P(B) = P(A \cap B) + P(A^c \cap B)$ [6]

1.4. The probability mass function of the discrete random variable X is given by

$$P(x) = \begin{cases} c \binom{4}{x} \binom{5}{5-x}, & x = 0,1,2,3,4 \\ 0, & \text{elsewhere} \end{cases}$$

1.4.1. Find the value of c [4]

1.4.2. Assuming $c = \frac{1}{126}$, find the cumulative distribution of X [4]

1.4.3. Find the median [2]

QUESTION 2 [10 Marks]

The results of a survey involving the use of sleepwear while travelling were listed as follows:

	Female	Male
Underwear	0.22	0.024
Nightgown	0.002	0.18
Nothing	0.16	0.018
Pajamas	0.102	0.073
T-shirt	0.046	0.088
Other	0.084	0.003

2.1. What is the probability that a traveller is a female who sleeps in the nightgown? [1]

2.2. What is the probability that a traveller is a female or sleeps in the pajamas? [2]

2.3. What is the probability that a traveller is a male if he sleeps in the pajamas or t-shirt? [4]

- 2.4. Assuming a traveller is male, what is the probability that he sleeps pajamas? [3]

QUESTION 3 [15 Marks]

- 3.1. Police plan to reinforce speed limits by using a radar traps at 4 different locations with the city of Windhoek. The radar traps at each of the locations L1, L2, L3, and L4 are operated 40%, 30%, 20%, and 30% of the time, and if a person who is speeding on his way to has a probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that he will receive a speeding ticket? [5]

Further, if the person received a speeding ticket on his way to work, what is the probability that he passed through the radar trap at L2? [5]

- 3.2 A diagnostic test for cancer is said to be 98% accurate if a person has the disease. Also, if a person does not have cancer, the test will report that he or she does not have it with probability 0.1. Only 0.1% has the disease in question. If a person is chosen at random from the population and diagnostic test indicates that he or she has cancer, what is the probability that he or she does, in fact, have cancer. [5]

QUESTION 4[20 Marks]

- 4.1. A large industrial firm purchase several new word-processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of wordprocessors, X , that are purchased each year has the following probability distribution:

X	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

If the cost on new wordprocessors at the end of this year is given by $12000 - 50X^2$, in Namibia Dollars,

- 4.1.1. how much can this firm expect to spend on new wordprocessors at the end of this year? [6]
- 4.1.2. find the variance of the number of wordprocessors that are purchased for this firm at the end of this year. [7]

4.1.3. find the coefficient of variation for the number of wordprocessors that are purchased for this firm at the end of this year. [4]

4.2. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem find

$$P(5 < X < 15) \quad [3]$$

QUESTION 5[25 Marks]

5.1. Let X be a binomial random with a probability mass function given by

$$f(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & \text{for } x = 0, 1, \dots, n \\ 0, & \text{elsewhere} \end{cases}$$

Show that $\sum_{x=0}^n f(x) = 1$. [5]

5.2. A multiple choice test consists of five questions with each question having four alternative answers, only one of which is correct. To pass you need at least four right answers. What is the probability that you pass? [4]

5.3. Accrotime is a manufacturer of quartz crystal watches. Accrotime researchers have shown that the watches have an average life of 28 months before certain electronic components deteriorate, causing the watch to become unreliable. The standard deviation of watch lifetimes is 5 months, and the distribution of lifetimes is normal.

5.3.1. If Accrotime guarantees a full refund on any defective watch for 2 years after purchase, what percentage of total production should the company expect to replace? [4]

5.3.2. If Accrotime does not want to make refunds on more than 12% of the watches it makes, how long should the guarantee period be (to the nearest month)? [5]

5.4. A secretary makes 2 errors per page, on average.

5.4.1. What is the probability that on the page he or she will make 4 or more errors? [5]

5.4.2. What is the expected number of errors in the next three pages? [2]

END OF QUESTION PAPER

Standard Normal Probabilities

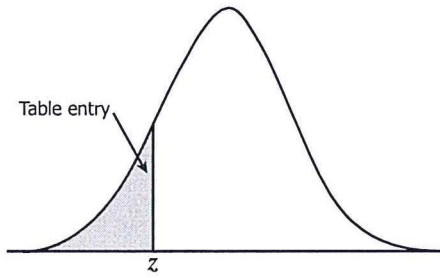


Table entry for z is the area under the standard normal curve to the left of z .

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

